LTH–896
BU-HEPP 11-01
CASPER 11-01
hep-ph/

Investigation of Quasi–Realistic Heterotic String Models with Reduced Higgs Spectrum

G. Cleaver*[1], A. Faraggi†[2], J. Greenwald‡[1], D. Moore§[1], K. Pechan ¶[3], E. Remkus‖[1] and T. Renner **[1]

[1] *CASPER, Department of Physics, Baylor University, Waco, TX, 76798-7316*
[2] *Department of Mathematical Sciences, University of Liverpool, Liverpool L69 7ZL*
[3] *Mitchell Institute, Dept. of Physics, Texas A&M, College Station, TX, 77843-4242*

Abstract

Quasi–realistic heterotic–string models in the free fermionic formulation typically contain an anomalous $U(1)$, which gives rise to a Fayet–Iliopolous term that breaks supersymmetry at the one–loop level in string perturbation theory. Supersymmetry is restored by imposing F– and D–flatness on the vacuum. In [14] we presented a three generation free fermionic standard–like model which did not admit stringent F– and D–flat directions, and argued that the all the moduli in the model are fixed. The particular property of the model was the reduction of the untwisted Higgs spectrum by a combination of symmetric and asymmetric boundary conditions with respect to the internal fermions associated with the compactified dimensions. In this paper we extend the analysis of free fermionic models with reduced Higgs spectrum to the cases in which the $SO(10)$ symmetry is left unbroken, or is reduced to the flipped $SU(5)$ subgroup. We show that all the models that we study in this paper do admit stringent flat directions. The only examples of models that do not admit stringent flat directions remain the standard–like models of ref. [14].

*Gerald_Cleaver@baylor.edu
†Faraggi@amtp.liv.ac.uk
‡Jared_Greenwald@baylor.edu
§Douglas_Moore1@baylor.edu
¶Kristen.Pechan@physics.tamu.edu
‖Erik_Remkus@baylor.edu
**Timothy_Renner@baylor.edu

1 Introduction

String theory provides a viable framework to probe the unification of gravity and the gauge interactions. Progress in this endeavor mandates the development of detailed phenomenological models as well as improved understanding of the mathematical structures that underly the theory. On the other hand the Standard Particle Model is compatible with all contemporary terrestrial and extra–terrestrial experimental data. Furthermore, the particle physics data is compatible with the hypothesis that the perturbative Standard Model remains unaltered up to the Planck scale, and that the particle spectrum is embedded in representations of a Grand Unified Theory (GUT). Most appealing in this context is $SO(10)$ unification in which each Standard Model (SM) generation is embedded in a single spinorial $\mathbf{16}$ representation.

Heterotic string theory naturally produces models that preserve the grand unified embedding of the Standard Model states [1]. Among the most realistic string models [2, 3, 4, 5, 6, 7, 8, 9] constructed to date are the heterotic–string models in the free fermionic formulation, which are related to $\mathbb{Z}_2 \times \mathbb{Z}_2$ toroidal orbifold compactifications [10]. In these models the $SO(10)$ GUT symmetry is broken directly at the string level, and include: the flipped $SU(5)$ string models [2]; the SM-like string models [3, 5, 7]; the Pati–Salam string models [4, 9]; and the Left–Right symmetric string models [8] (LRS). Generically, string vacua contain numerous fields beyond the SM states and these must be given sufficiently heavy mass. Heterotic string models necessarily also contain exotic fractionally charged states that are severely constrained by observations. These can be given mass on the order of the Planck scale by Standard Model singlet VEVs [7], or may be entirely absent from the massless spectrum [9]. The free fermionic heterotic string models produced the first known string models in which the matter content of the effective low energy observable sector consists solely of the Minimal Supersymmetric Standard Model (MSSM) [7]. The issue of the supersymmetric moduli space and supersymmetry breaking remains one of the important open issues in string phenomenology.

The relation of free fermion models to $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds mandates the production of three untwisted pairs of vectorial 10 representations, which decompose as $[\mathbf{5} \oplus \bar{\mathbf{5}}]$ under $SU(5)$ and as $\left[(\bar{\mathbf{3}}, \mathbf{1})_{1/3} + (\mathbf{1}, \mathbf{2})_{-1/2}\right] \oplus \left[(\mathbf{3}, \mathbf{1})_{-1/3} + (\mathbf{1}, \mathbf{2})_{1/2}\right]$ under the Standard Model $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group. This presents an excess beyond the two electroweak Higgs doublets that are required in the MSSM. In the Pati–Salam and Standard–like string models either the color triplets or electroweak Higgs doublets from each pair may be projected from the massless spectrum by asymmetric/symmetric boundary conditions with respect to a given twisted plane [12]. Assignment of symmetric and asymmetric boundary conditions in two different basis vectors results in the projection of both color triplets and electroweak doublets and therefore reduces the spectrum without the need to give them heavy mass by singlet VEVs [13, 14]. In this manner entire $\mathbf{10}$ vectorial pairs can be projected. An additional consequence of the symmetric plus asymmetric assignment is that untwisted

$SO(10)$ singlet states are projected as well and the supersymmetric moduli space is constrained [13, 14].

Standard–like string models that employ the symmetric plus asymmetric boundary condition assignment to reduce the light Higgs spectrum were constructed in refs. [13, 14]. The model constructed in ref. [13] did not admit flat directions that preserve the Standard Model (SM) gauge group. This gave rise to the possibility that the model does not admit supersymmetric flat directions at all. This would be a very interesting situation as it would indicate that supersymmetry is broken perturbatively in such models. In ref. [14] this question was examined further by analyzing the so–called stringent flat directions. It was demonstrated that there exists SM–like string models with reduced untwisted Higgs spectrum, which do not admit stringent flat directions to any order of non-renomalizable superpotential term. This outcome is in contrast to the case of all other quasi–realistic free fermionic models, which have been shown to admit such stringent flat directions.

The question therefore arises whether the absence of stringent flat directions (see Figure 1) in free fermion heterotic–string models with reduced Higgs spectrum is particular to the SM–like models, and whether similar results arise in cases in which the $SO(10)$ subgroup is broken to an alternative subgroup. In this paper we examine this question by analyzing the D- and F–flat directions in four models. Three in which the $SO(10)$ symmetry is broken to the the flipped $SU(5)$ subgroup and one in which the $SO(10)$ symmetry remains unbroken. We show that all of these cases admit stringent supersymmetric flat directions. Hence, SM–like models with reduced Higgs spectrum remain the only examples in which stringent flat directions have been shown not to exist.

Our paper is organized as follows: Section 2 contains a review of Free Fermionic Models construction using the NAHE set and additional boundary vectors to control gauge and SUSY breaking. In Section 3, we review the process of flat directions used with EFTs as pertaining to Free Fermionic strings. Here there is a further discussion on stringent F-flat directions. The string models in question are reviewed in Sections 4 and 5, where we list the boundary vectors pertaining to each model as well as summarize the flat directions. We conclude our discussion in Section 6 and make comment on the impact of our results on future studies of the Free Fermionic Heterotic Landscape.

2 Free Fermionic Models

In this section we briefly review the construction and structure of the free fermionic standard like models. The notation and further details of the construction of these models are given elsewhere [3, 5, 15, 7, 8, 13]. In the free fermionic formulation of the heterotic string in four dimensions [16] all the world–sheet degrees of freedom, required to cancel the conformal anomaly, are represented in terms of free fermions propagating on the string world–sheet. In the light–cone gauge the world–sheet field content consists of two transverse left– and right–moving space–time coordinate bosons, $X^\mu_{1,2}$ and $\bar{X}^\mu_{1,2}$, and their left–moving fermionic superpartners, $\psi^\mu_{1,2}$, and additional 62 purely internal Majorana–Weyl fermions, of which 18 are left–moving and 44 are right–moving. The models are constructed by specifying the phases produced by the world–sheet fermions when transported along the torus' non–contractible loops

$$f \to -e^{i\pi\alpha(f)}f, \quad \alpha(f) \in (-1, 1]. \tag{2.1}$$

Each model corresponds to a particular choice of fermion phases consistent with modular invariance and is generated by a set of basis vectors describing the transformation properties of the 64 world–sheet fermions. The physical spectrum is obtained by applying the generalized GSO projections. The GSO projection is administered through the following equations:

$$\mathbf{V}_j \cdot \mathbf{Q}_\alpha = \left(\sum_i k_{i,j} a_i\right) + s_j \quad (mod\ 2) \tag{2.2}$$

$$\boldsymbol{\alpha}(f) = \sum_{i=1}^{D} a_i \mathbf{V}_i \ \ (mod\ 2), \quad \alpha_i \in \{\mathbb{Z}|0 \le \alpha \le N_i - 1\} \tag{2.3}$$

$$\mathbf{Q}(f) \equiv \frac{1}{2}\boldsymbol{\alpha}(f) + \mathbf{F}(f) \tag{2.4}$$

where $\alpha(f)$ is the boundary vector defined by eq. 2.1 and is expanded in terms of basis vectors, $\mathbf{V}$. For all 64 components of $\mathbf{V}_i$, N_i is the smallest positive integer such that $N_i\mathbf{V}_i = 0 \ \ (mod\ 2)$. $\mathbf{F}$ is the fermion number operator and is equal to $\{0, \pm 1\}$ for non-periodic fermions and $\{0, -1\}$ for periodic. After construction of the string model, the low energy effective field theory is obtained by S–matrix elements between external states [17].

The boundary condition basis defining a typical realistic free fermionic heterotic string model is constructed in two stages. The first stage consists of the NAHE set, which is a set of five boundary condition basis vectors, $\{\mathbf{1}, S, b_1, b_2, b_3\}$ [15]. The gauge group, after imposing the GSO projections induced by the NAHE set, is $SO(10) \times SO(6)^3 \times E_8$, with $N = 1$ supersymmetry. The NAHE set divides the internal world–sheet fermions in the following way: $\bar{\phi}^{1,\cdots,8}$ generate the hidden E_8 gauge group, $\bar{\psi}^{1,\cdots,5}$ generate the $SO(10)$ gauge group, while $\{\bar{y}^{3,\cdots,6}, \bar{\eta}^1\}$, $\{\bar{y}^1, \bar{y}^2, \bar{\omega}^5, \bar{\omega}^6, \bar{\eta}^2\}$ and $\{\bar{\omega}^{1,\cdots,4}, \bar{\eta}^3\}$ generate the three horizontal $SO(6)$ symmetries. The left–moving $\{y, \omega\}$ states are divided into $\{y^{3,\cdots,6}\}$, $\{y^1, y^2, \omega^5, \omega^6\}$, $\{\omega^{1,\cdots,4}\}$, while χ^{12}, χ^{34}, χ^{56} generate the left–moving $N = 2$ world–sheet supersymmetry.

The second stage of the basis construction consists of adding to the NAHE set three additional boundary condition basis vectors. These additional basis vectors reduce the number of generations to three chiral generations, one from each of the sectors b_1, b_2 and b_3, and simultaneously break the four dimensional gauge group.

The assignment of boundary conditions to $\{\bar{\psi}^{1,\cdots,5}\}$ breaks $SO(10)$ to one of its subgroups. Similarly, the hidden E_8 symmetry is broken to one of its subgroups. The flavor $SO(6)^3$ symmetries in the NAHE–based models are always broken to flavor $U(1)$ symmetries, as the breaking of these symmetries is correlated with the number of chiral generations. Three such $U(1)_j$ symmetries are always obtained in the NAHE based free fermionic models from the subgroup of the observable E_8, which is orthogonal to $SO(10)$. These are produced by the world–sheet currents $\bar{\eta}^j\bar{\eta}^{j^*}$ $(j = 1, 2, 3)$, which are part of the Cartan sub–algebra of the observable E_8. Additional unbroken $U(1)$ symmetries, denoted typically by $U(1)_j$ $(j = 4, 5, ...)$, arise by pairing two real fermions from the sets $\{\bar{y}^{3,\cdots,6}\}$, $\{\bar{y}^{1,2}, \bar{\omega}^{5,6}\}$ and $\{\bar{\omega}^{1,\cdots,4}\}$. The final observable gauge group depends on the number of such pairings. Alternatively, a left–moving real fermion from the sets $\{y^{3,\cdots,6}\}$, $\{y^{1,2}, \omega^{5,6}\}$ and $\{\omega^{1,\cdots,4}\}$ may be paired with its respective right–moving real fermion to form an Ising model operator, in which case the rank of the right–moving gauge group is reduced by one. The reduction of untwisted electroweak Higgs doublets crucially depends on the pairings of the left– and right–moving fermions from the set $\{y, \omega|\bar{y}, \bar{\omega}\}^{1\cdots 6}$.

Subsequent to constructing the basis vectors and extracting the massless spectrum, the analysis of the free fermionic models proceeds by calculating the superpotential. The cubic and higher-order terms in the superpotential are obtained by evaluating the correlators

$$A_N \sim \langle V_1^f V_2^f V_3^b \cdots V_N^b \rangle, \tag{2.5}$$

where V_i^f (V_i^b) are the fermionic (scalar) components of the vertex operators, using the rules given in [17].

3 Flat Directions of Free Fermions

A common feature of many of the quasi–realistic free fermionic heterotic–string models is the existence of "anomalous" $U(1)$'s generated from compactification or from gauge group breaking [19]. If multiple $U(1)$'s are anomalous, the anomaly can always be rotation into a single $U(1)_A$ while the remaining orthogonal $U(1)$'s all become non-anomalous. This anomalous $U(1)_A$ is broken by the Green–Schwarz–Dine–Seiberg–Witten mechanism [20] in which a potentially large Fayet–Iliopoulos D–term ξ is generated by the VEV of the dilaton field. (For the class of free fermionic models under investigation $\xi > 0$.) Such a D–term would, in general, break supersymmetry, unless there is a direction $\hat{\phi} = \sum \alpha_i \phi_i$ in the scalar potential for which $\sum Q_A^i |\alpha_i|^2$ is of opposite sign to ξ and that is D–flat with respect to all the non–anomalous gauge symmetries, as well as F–flat. If such a direction exists, it will acquire a VEV, canceling the Fayet–Iliopoulos ξ–term, restoring supersymmetry and stabilizing the vacuum.

The D–term is formally defined as:

$$D_a^\alpha \equiv K_\alpha + \sum_m \phi_m^\dagger T_a^\alpha \phi_m, \tag{3.1}$$

where T_a^α is a generator and ϕ_m is a representation of the group, $\mathcal{G}_\alpha$ while the K–terms contain fields Φ_i like squarks, sleptons and Higgs bosons whose VEVs should vanish at this scale for good phenomenology. When the ϕ_i are singlets of all the non–Abelian gauge groups in a model, the set of D- and F-flat constraints are then given by

$$\langle D_A \rangle = \langle D_\alpha \rangle = 0 \ ; \quad \langle F_i \equiv \frac{\partial W}{\partial \Phi_i} \rangle = 0 \ \ ; \tag{3.2}$$

$$D_A = \left[K_A + \sum Q_A^k |\phi_k|^2 + \xi \right] \ \ ; \tag{3.3}$$

$$D_\alpha = \left[K_\alpha + \sum Q_\alpha^k |\phi_k|^2 \right] \ , \ \alpha \neq A \ \ ; \tag{3.4}$$

$$\xi = \frac{g^2 (\mathrm{Tr} Q_A)}{192\pi^2} M_{\mathrm{Pl}}^2 \ \ ; \tag{3.5}$$

where χ_k are the fields which acquire VEVs of order $\sqrt{\xi}$. The Φ_i is the superfield formed from the spacetime scalar ϕ_i and the chiral spin-1/2 fermion ψ_m, while Q_A^k and Q_α^k denote the anomalous and non–anomalous charges and $M_{\mathrm{Pl}} \approx 2 \times 10^{18}$ GeV denotes the reduced Planck mass. The solution (*i.e.* the choice of fields with non–vanishing VEVs) to the set of equations (3.2)–(3.4), though nontrivial, is not unique. Therefore in a typical model there exist a moduli space of solutions to the F- and D-flatness constraints, which are supersymmetric and degenerate in energy [21]. Much of the study of the superstring model phenomenology (as well as non–string supersymmetric models [22]) involves the analysis and classification of these flat directions. The methods for this analysis in string models have been systematized in [23, 24, 7, 25].

In general it is assumed in the literature that in a given string model there should exist a supersymmetric solution to the F- and D- flatness constraints. The simpler type of solutions utilize only fields that are singlets of all the non–Abelian groups in a given model (type I solutions). More involved solutions (type II) that utilize non–Abelian fields have also been considered [25], as well as inclusion of non–Abelian fields in systematic methods of analysis [25]. The general expectation that a given model admits a supersymmetric solution arises from analysis of supersymmetric point quantum field theories. In these cases it is known that if supersymmetry is preserved at the classical level, *i.e.* tree–level in perturbation theory, then there exist index theorems that forbid supersymmetry breaking at the perturbative quantum level [26]. Therefore in point quantum field theories supersymmetry breaking may only be induced by non–perturbative effects [27].

The issue of supersymmetry breaking in the string models that were constructed in refs. [13, 14] merits further investigation. The aim of these models was to construct SM–like string models with reduced untwisted Higgs spectrum. This was achieved by utilizing asymmetric boundary conditions in a basis vector that does not break the $SO(10)$ symmetry [13]. The consequence is that the entire vectorial **10** representation from the corresponding plane is projected out. The alternative is to utilize a combination of symmetric and asymmetric boundary conditions in basis vectors that break the $SO(10)$ symmetry to the Pati–Salam gauge group [14]. An unforeseen consequence of the Higgs reduction mechanism of refs. [13, 14] was the simultaneous projection of untwisted $SO(10)$ singlet fields. Consequently, the moduli space of supersymmetric flat solutions is vastly reduced. In ref. [13] it was concluded that the model under investigation there does not contain supersymmetric flat directions that do not break some of the SM symmetries.

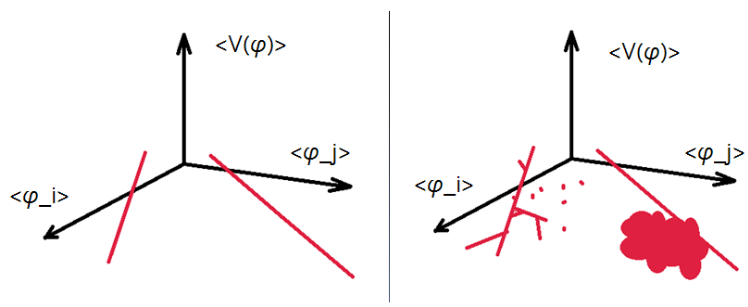

Figure 1: On the left, one will find general characteristics of a model by investigating the stringent flat directions ('root' directions). It seems that every all-order flat direction (right) is attached to or near a 'root' direction found via the stringency requirement.

For a given set of flat D-term VEVs, an F-term will break supersymmetry if its expectation value is non-zero. The scale of supersymmetry breaking is a function of the number of fields in the lowest-order, non-zero component of an F-term. Specifically, the lower the number of fields, the higher the energy scale of SUSY breaking. For example, a surviving second order component of an F-term breaks SUSY at the string/Planck scale, whereas a 17^{th} order component breaks as needed around the 100 GeV to 1 TeV scale. This SUSY-breaking energy scale is decreased approximately 10 GeV per order increase of the non-zero F-term components.

In ref. [14] the issue of superymmetry breaking was investigated further. It was shown that the model studied in [14] does not have D-flat directions that can be proven to be F-flat to all order, other than through order-by-order analysis. That is, there do not appear to be any D-flat directions with *stringent* F-flatness (as defined in [7, 11, 28]). The analysis of the flat directions included all the fields in the string model, *i.e.* SM singlet states as well as SM charged states. The model of ref. [14]

therefore does not contain a D–flat direction that is also stringently F–flat to all order of non–renormalizable terms. The model may of course still admit non-stringent flat directions that rely on cancellations between superpotential terms. However, past experience suggests that non–stringent flat directions can only hold order by order, and are not maintained to all orders [29, 8]. This is a key difference between the string theory case, in which heavy string modes generate an infinite tower of terms, versus the field theory case in which heavy modes are not integrated out. It was therefore speculated that in this case supersymmetry is not exact, but is in general broken at some order.

The SM–like string models presented in refs [13, 14] contain three chiral generations, charged under the SM gauge group and with the canonical $SO(10)$ embedding of the weak–hypercharge, one pair of untwisted electroweak Higgs doublets and a cubic level top–quark Yukawa coupling. These string models therefore share some of the phenomenological characteristics of the quasi–realistic free fermionic string models. It may therefore represent an example of a quasi–realistic string model, in which supersymmetry is broken due to the existence of the heavy string modes. The utilization of asymmetric boundary conditions in these models entails that the geometrical moduli in these models are projected out [30]. The Higgs reduction mechanism of refs. [13, 14] further reduces the supersymmetric moduli space, and possibly fixes it completely, while the hidden sector satifies the conditions for the dilaton race–track stabilization mechanism [37, 14]. It is noted that imposing more restrictive phenomenological constraints is correlated with further reduction of the moduli space. The question of interest is therefore whether the restriction of the supersymmetric moduli space is specific to the case of standard–like models. In the next section we investigate the Higgs reduction mechanism in a model with an unbroken $SO(10)$ symmetry and in three flipped $SU(5)$ models.

4 The String Models

In this section we present the $SO(10)$ and flipped $SU(5)$ string models that utilize the Higgs reduction mechanism of [13, 14]. We first review how the Higgs doublet–triplet splitting mechanism operates in the free fermionic models. For concreteness we illustrate how the mechanism operates in the model of ref. [3].

4.1 Higgs Doublet–Triplet Splitting

An example of a free fermionic SM–like model is given in Table 4.1[1] .

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...,8}$
b_4	1	1	0	0	1 1 1 1 1	1	0	0	0 0 0 0 0 0 0 0
α	1	0	0	1	1 1 1 0 0	1	1	0	1 1 1 1 0 0 0 0
β	1	0	1	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1 1 $\frac{1}{2}$ 0 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 0

	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^6$	$y^2\bar{y}^2$	$\omega^5\bar{\omega}^5$	$\bar{y}^1\bar{\omega}^6$	$\omega^1\omega^3$	$\omega^2\bar{\omega}^2$	$\omega^4\bar{\omega}^4$	$\bar{\omega}^1\bar{\omega}^3$
b_4	1	0	0	1	0	0	1	0	0	0	1	0
α	0	0	0	1	0	1	0	1	1	0	1	0
β	0	0	1	1	1	0	0	1	0	1	0	0

(4.1)

The Higgs doublet–triplet splitting operates as follows [12]. The Neveu–Schwarz sector gives rise to three fields in the **10** representation of $SO(10)$. These contain the Higgs electroweak doublets and color triplets. Each of those is charged with respect to one of the horizontal $U(1)$ symmetries $U(1)_{1,2,3}$. Each one of these multiplets is associated, by the horizontal symmetries, with one of the twisted sectors, b_1, b_2 and b_3. The doublet–triplet splitting results from the boundary condition basis vectors which break the $SO(10)$ symmetry to $SO(6) \times SO(4)$. We can define a quantity Δ_i in these basis vectors which measures the difference between the boundary conditions assigned to the internal fermions from the set $\{y, w|\bar{y}, \bar{\omega}\}$ and which are periodic in the vector b_i,

$$\Delta_i = |\alpha_L(\text{internal}) - \alpha_R(\text{internal})| = 0, 1 \quad (i = 1, 2, 3). \tag{4.2}$$

If $\Delta_i = 0$ then the Higgs triplets, D_i and $\bar{D}_i$, remain in the massless spectrum while the Higgs doublets, h_i and $\bar{h}_i$, are projected out and the opposite occurs for $\Delta_i = 1$.

The rule in Eq. (4.2) is a generic rule that operates in free fermionic models. The model of eq. (4.1) illustrates this rule. In this model the basis vector that breaks $SO(10)$ symmetry to $SO(6) \times SO(4)$ is α and, with respect to α, $\Delta_1 = \Delta_2 = \Delta_3 = 1$. Therefore, this model produces three pairs of electroweak Higgs doublets from the Neveu–Schwarz sector, h_1, $\bar{h}_1$ h_2, $\bar{h}_2$ and h_3, $\bar{h}_3$, and all the untwisted color triplets are projected out. Note also that the vector basis b_4 is symmetric with respect to the internal fermions that are periodic in the vectors b_i, $i = 1, 2, 3$ and, therefore, does not project out the fields in the **10** representation of $SO(10)$.

Another possibility is to construct models in which both the Higgs color triplets and electroweak doublets from the Neveu–Schwarz sector are projected out by the GSO projections. This is a viable possibility as we can choose for example

$$\Delta_j^{(\alpha)} = 1 \text{ and } \Delta_j^{(\beta)} = 0,$$

[1]Only the boundary condition basis vectors beyond the NAHE–set are displayed.

where $\Delta^{(\alpha,\beta)}$ are the projections due to the basis vectors α and β respectively. This is desirable as the number of Higgs representations, which generically appear in the massless spectrum, is larger than what is allowed by the low energy phenomenology. The model shown in eq. (4.3) illustrates this possibility

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...,8}$
α	0	0	0	0	1 1 1 0 0	1	0	0	1 1 0 0 0 0 0 0
β	0	0	0	0	1 1 1 0 0	0	1	0	0 0 1 1 0 0 0 0
γ	0	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 0 0 0 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$
α	1	0	0	1	0	0	1	1	0	0	1	1
β	0	0	1	1	1	0	0	1	0	1	0	1
γ	0	1	0	0	0	1	0	0	1	0	0	0

(4.3)

Both the basis vectors α and β break the $SO(10)$ symmetry to $SO(6) \times SO(4)$ and the basis vector γ breaks it further to $SU(3) \times U(1)_C \times SU(2) \times U(1)_L$. The basis vector α is symmetric with respect to the sector b_1 and asymmetric with respect to the sectors b_2 and b_3, whereas the basis vector β is symmetric with respect to b_2 and asymmetric with respect to b_1 and b_3. As a consequence of these assignments and of the string doublet–triplet splitting mechanism discussed above, both the untwisted Higgs color triplets and electroweak doublets, with leading coupling to the matter states from the sectors b_1 and b_2, are projected out by the generalized GSO projections. At the same time the untwisted color Higgs triplets that couple at leading order to the states from the sector b_3 are projected out, whereas the untwisted electroweak Higgs doublets remain in the massless spectrum. Due to the asymmetric boundary conditions in the sector γ with respect to the sector b_3, the leading Yukawa coupling is that of the up–type quark from the sector b_3 to the untwisted electroweak Higgs doublet [6]. Hence, the leading Yukawa term is that of the top quark and only its mass is characterized by the electroweak VEV [6]. The lighter quarks and leptons couple to the light Higgs doublet through higher order non-renormalizable operators that become effective renormalizable operators by the VEVs that are used to cancel the anomalous $U(1)_A$ D–term equation [6]. The novelty in the construction of (4.3) is that the reduction of the untwisted Higgs spectrum is obtained by the choice of the boundary condition basis vectors in eq. (4.3), without resorting to analysis of supersymmetric flat directions, whereas in other models it is obtained by the choice of flat directions and analysis of the superpotential [18].

However, the surprising result was that the model does not seem to admit a stringent supersymmetric solution [14]. This appeared to be a consequence of the reduction of untwisted singlet states (denoted by Φ_i and ϕ_i in that model and in all models presented here), simultaneous with the untwisted Higgs reduction, imposed by the asymmetric/symmetric boundary conditions. We next turn to examine this issue in models in which the $SO(10)$ symmetry remains unbroken, or is broken to the flipped $SU(5)$ subgroup.

4.2 An $SO(10)$ Model

In the previous section we discussed the Higgs reduction mechanism of ref. [13, 14]. We noted that the reduction is achieved by utilizing symmetric and asymmetric boundary conditions in two separate basis vectors. However, the reduction can be obtained if we assign asymmetric boundary conditions in a basis vector that does not break the $SO(10)$ symmetry. The model in Table 1 is an example of such a model.

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar\psi^{1,\dots,5}$	$\bar\eta^1$	$\bar\eta^2$	$\bar\eta^3$	$\bar\phi^{1,\dots,8}$
b_4	0	0	0	0	1 1 1 1 1	1	0	0	0 0 0 0 0 0 0 0
b_5	0	0	0	0	1 1 1 1 1	0	1	0	0 0 0 0 0 0 0 0
b_6	0	0	0	0	1 1 1 1 1	1	1	0	1 1 0 0 0 0 0 0
2γ	0	0	0	0	1 1 1 1 1	1	1	1	1 0 1 1 1 0 0 0

	$y^{3\dots6}$	$\bar y^{3\dots6}$	$y^{1,2}$ $\omega^{5,6}$	$\bar y^{1,2}$ $\bar\omega^{5,6}$	$\omega^{1\dots4}$	$\bar\omega^{1\dots4}$
b_4	1 0 0 1	1 0 0 1	0 0 0 1	1 0 1 1	0 0 1 0	0 1 1 1
b_5	0 0 1 0	1 0 1 1	1 0 1 0	1 0 1 0	1 0 0 0	1 1 0 1
b_6	0 1 0 0	0 1 0 0	0 1 0 0	0 1 0 0	0 1 0 1	0 0 0 0
2γ	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0

Table 1: Additional boundary vectors for the $SO(10)$ model

We note that in tables 1 and 2 vectors that do not break the $SO(10)$ symmetry are denoted by b_j with $j \geq 4$. Basis vectors that break the $SO(10)$ symmetry in the fliipped $SU(5)$ models below will be denoted by Greek letters. The last vector in tables 1 and 2 is denoted as 2γ to adhere with the notation used in the quasi–realistic free fermionic models [10].

The model defined by Tables 1 and 2 contain three spinorial **16** representations of the $SO(10)$ GUT group. All untwisted vectorial **10** representations are projected out. The asymmetric boundary conditions in b_4 with respect to b_2 and b_3 projects out the corresponding vectorial representations, whereas the asymmetric boundary condition in b_5 with respect to b_1 project out the remaining untwisted $SO(10)$ vectorial representations. The twisted sectors $b_1 + b_4$ and $b_1 + b_2 + b_4 + b_5$ give rise to $SO(10)$ vectorial representations and these are the sources of the two pairs of SM Higgs: $h_1(\bar h_1)$ and $h_2(\bar h_2)$.

We note that the model in Tables 1 and 2 is not phenomenologically realistic as it does not contain the heavy Higgs representations needed to break the $SO(10)$

$$
\begin{array}{cc}
 & \begin{matrix} \mathbf{1} & S & b_1 & b_2 & b_3 & b_4 & b_5 & 2\gamma & \delta \end{matrix} \\
\begin{matrix} \mathbf{1} \\ S \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ 2\gamma \\ \delta \end{matrix} &
\begin{pmatrix}
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1
\end{pmatrix}
\end{array}
$$

Table 2: GSO K–Matrix for the $SO(10)$ model. The K_{ij} components are labeled by column 'j' and row 'i'. All other GSO K–matrix tables follow the same pattern.

symmetry to the SM gauge group. Our interest in this model here is to study the existence and characteristic of the (all-order) stringent flat directions. Namely, do they exist, and do they preserve the SM (within the $SO(10)$) gauge group? This question will be discussed in section 5. The states and gauge charges for this model can be found in Table 21 located in Appendix A.

4.3 Flipped $SU(5)$ Models

Some potentially more realistic models are those with flipped $SU(5)$ observable sectors. This are listed below in sections: 4.3.1, 4.3.2 and 4.3.3.

4.3.1 Flipped $SU(5)$ Model 1

This flipped $SU(5)$ model has additional boundary vectors given in Table 3 and a GSO projection matrix shown in Table 4. The states and gauge charges for this model can be found in Table 22 located in Appendix A. This model is completely lacking in both untwisted and twisted Higgs.

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\dots,8}$
b_4	0	0	0	0	1 1 1 1 1	1	0	0	0 0 0 0 0 0 0 0
b_5	0	0	0	0	1 1 1 1 1	1	0	0	0 0 0 0 0 0 0 0
γ	0	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 0 0 0 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

	$y^{3\dots6}$	$\bar{y}^{3\dots6}$	$y^{1,2}\ \omega^{5,6}$	$\bar{y}^{1,2}\ \bar{\omega}^{5,6}$	$\omega^{1\dots4}$	$\bar{\omega}^{1\dots4}$
b_4	1 0 0 1	1 0 0 1	0 0 0 1	1 0 1 1	0 0 1 0	0 1 1 1
b_5	0 0 1 0	1 0 1 1	1 0 1 0	1 0 1 0	1 0 0 0	1 1 0 1
γ	0 1 0 0	0 1 0 0	0 1 0 0	0 1 0 0	0 1 0 1	0 0 0 0

Table 3: Additional boundary vectors for flipped $SU(5)$ model #1

$$
\begin{array}{c}
 \\ \mathbf{1} \\ S \\ \\ b_1 \\ b_2 \\ b_3 \\ \\ b_4 \\ b_5 \\ \gamma
\end{array}
\begin{array}{c}
\begin{array}{cccccccc}
\mathbf{1} & S & b_1 & b_2 & b_3 & b_4 & b_5 & \gamma
\end{array}\\
\left(\begin{array}{cccccccc}
0 & 0 & 1 & 1 & 1 & 0 & 0 & -\frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & \\
1 & 1 & 1 & 1 & 1 & 0 & 0 & -\frac{1}{2} \\
1 & 1 & 1 & 1 & 1 & 0 & 1 & \frac{1}{2} \\
1 & 1 & 1 & 1 & 1 & 1 & 0 & -\frac{1}{2} \\
 & & & & & & & \\
0 & 0 & 1 & 1 & 0 & 0 & 1 & -\frac{1}{2} \\
0 & 0 & 1 & 0 & 1 & 1 & 0 & \frac{1}{2} \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & -\frac{1}{2}
\end{array}\right)
\end{array}
$$

Table 4: GSO K-Matix for Flipped $SU(5)$ Model #1

4.3.2 Flipped $SU(5)$ model 2

This flipped $SU(5)$ model has additional boundary vectors given in Table 5 and a GSO projection matrix shown in Table 6. The states and gauge charges for this model can be found in Table 23 located in Appendix A. This model has two untwisted Higgs pairs and three more pairs of twisted Higgs.

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,\ldots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\ldots,8}$
b_4	1	0	0	0	1 1 1 1 1	0	1	0	0 0 0 0 0 0 0 0
b_5	1	0	1	0	1 1 1 1 1	1	0	0	0 0 0 0 0 0 0 0
γ	1	0	0	1	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

	$y^{3\ldots6}$	$\bar{y}^{3\ldots6}$	$y^{1,2}$ $\omega^{5,6}$	$\bar{y}^{1,2}$ $\bar{\omega}^{5,6}$	$\omega^{1\ldots4}$	$\bar{\omega}^{1\ldots4}$
b_4	1 0 0 1	0 0 0 0	0 0 1 0	1 0 1 1	0 0 0 1	0 0 0 1
b_5	0 0 0 0	1 0 0 1	1 0 1 1	0 0 1 0	0 1 0 0	0 1 0 0
γ	0 1 0 0	1 1 0 1	0 1 0 0	0 1 0 0	1 0 1 0	0 0 0 0

Table 5: Additional boundary vectors for $SU(5)$ model #2

$$
\begin{array}{cc}
 & \begin{array}{cccccccc} \mathbf{1} & S & b_1 & b_2 & b_3 & b_4 & b_5 & \gamma \end{array} \\
\begin{array}{c} \mathbf{1} \\ S \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ \gamma \end{array} &
\begin{pmatrix}
0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 0 & 0 & \frac{1}{2} \\
1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & -\frac{1}{2} \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & \frac{1}{2}
\end{pmatrix}
\end{array}
$$

Table 6: GSO K-Matix for Flipped $SU(5)$ Model #2

4.3.3 Flipped $SU(5)$ Model 3

This flipped $SU(5)$ model has additional boundary vectors given in Table 7 and a GSO projection matrix shown in Table 8. The states and gauge charges for this model can be found in Table 24 located in Appendix A.

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...,8}$
b_4	1	1	0	0	1 1 1 1 1	0	1	0	0 0 0 0 0 0 0 0
b_5	1	0	1	0	1 1 1 1 1	1	0	0	0 0 0 0 0 0 0 0
γ	1	0	0	1	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 0 0

	$y^{3...6}$	$\bar{y}^{3...6}$	$y^{1,2}$ $\omega^{5,6}$	$\bar{y}^{1,2}$ $\bar{\omega}^{5,6}$	$\omega^{1...4}$	$\bar{\omega}^{1...4}$
b_4	1 0 0 1	0 0 0 0	0 0 1 0	1 0 1 1	0 0 0 1	0 0 0 1
b_5	0 0 0 0	1 0 0 1	1 0 1 1	0 0 1 0	0 1 0 0	0 1 0 0
γ	0 1 0 0	1 1 0 1	0 0 0 0	1 0 0 1	1 1 1 0	0 1 0 0

Table 7: Additional boundary vectors for $SU(5)$ model #3

$$
\begin{array}{c}
\\ \mathbf{1} \\ S \\ \\ b_1 \\ b_2 \\ b_3 \\ \\ b_4 \\ b_5 \\ \gamma
\end{array}
\begin{array}{c}
\begin{array}{cccccccc} \mathbf{1} & S & b_1 & b_2 & b_3 & b_4 & b_5 & \gamma \end{array} \\
\begin{pmatrix}
0 & 0 & 1 & 1 & 1 & 0 & 0 & -\frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
 & & & & & & & \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & -\frac{1}{2} \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & \frac{1}{2} \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
 & & & & & & & \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & \frac{1}{2} \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & \frac{1}{2} \\
1 & 1 & 1 & 1 & 0 & 1 & 1 & 1
\end{pmatrix}
\end{array}
$$

Table 8: GSO K-Matix for Flipped $SU(5)$ Model #3

This last model produces three generations from the sectors b_1, b_2 and b_3 and one light Higgs pair from the untwisted sector. The sectors b_4 and b_5 give rise to heavy Higgs states that are needed to break the $SU(5)$ gauge symmetry to the Standard Model gauge group. The model admits a cubic level top quark Yukawa coupling of order one. In this respect the model reproduces many of the features of other quasi–realistic flipped $SU(5)$ string models. Its distinctive characteristic is that two of the untwisted Higgs pairs are projected out due to the asymmetric boundary conditions in b_4 and b_5 with respect to b_1 and b_2.

5 Flat directions

Here we list the results of the flat direction analysis. These lists are by no means exhaustive but are presented just as a "proof of concept". The $SO(10)$ model has D–flat directions found in Table 9 and all-order stringent F–flat directions in Table 10. For this $SO(10)$ model it was necessary only to investigate type I flat directions (those formed with fields that are singlets under both the observable and hidden sector gauge groups). This is, of course, the preferred option as it is both simpler and computationally less-expensive. The all-order stringent flat directions we found possessed only VEVs of singlets fields and since the SM model is completely contained within $SO(10)$ that means that the SM remains unbroken here.

Similarly, those flat directions investigated in flipped $SU(5)$ model #2, were also of type I. The D-flat directions for this model can be found in Table 15 and the F-flat table can be found in Table 16. We note that the all-order stringent flat directions in Table 16 contain VEVs for one or more generations of anti-electrons. Thus the SM hypercharge is broken in this model. Interestingly, this model contains an exotic negatively-charged electron singlet forming a vector-pair with the corresponding anti-electron (see Table 23).

Both flipped $SU(5)$ models #1 and #3, however, lacked type I stringent flat directions and required an investigation of those fields which are charged under some non-Abelian gauge representation. For model #1, the failure to produce singlet D-flat directions that can solely cancel the Fayet-Iliopoulos term came as a result of all possible directions having an anomalous charge of zero (see Table 11). This condition requires the search for type II directions with negative anomalous charge. This search is a nonlinear process ([14],[31]) and, for the purposes of our investigation, the use of stringency requirements offered a much quicker, and still fruitful, search. The net effect of stringent flatness is that: (1.) at least two fields (including a given field appearing twice) must not take on a VEV or (2.) self-cancellation between components of a non-abelian field must occur. Additionally for model #1, we found (as can be seen in Table 12) non-Abelian D-flat directions that were negative. However, these directions failed to yield F-flat directions by themselves; this necessitated the mixing of both singlet and non-Abelian D-flat directions in order to find all-order stringent F-flatness. These mixed D-flat directions can be seen in Table 13.

The speed increase from stringent F-flat direction searches results from only needing to investigate a finite set of potentially dangerous superpotential terms which determine all-order flatness. The answer to whether or not these terms are dangerous (i.e. they actually exist) comes from the fact that these models are derived from string theory and, therefore, the gauge invariant terms need to also obey Ramon/Neveu-Schwarz (RNS) worldsheet charge conservation. This then makes string-derived models more constrained than the usual QFT-derived ones. These all-order RNS rules can be found in [25].

The D- and F-flat tables for flipped $SU(5)$ model #1 can be found in Tables

11–14. Here the $SU(5)$ gauge group remains unbroken. However, hypercharge is most likely broken in this model since in flipped $SU(5)$ there is a contribution to hypercharge from an external $U(1)$ and all extra $U(1)'_i$ appear to be carried by the singlet and/or non-Abelian states acquiring VEVs in the all-order stringent F-flat directions (see Tables 14 and 22). Additionally the hidden sector $SU(4)$ is broken by the $\bar{H}_i$ while the $SO(10)_{hidden}$ remains unbroken.

For flipped $SU(5)$ model #3, the D- and F-flat tables can be found in Tables 17–19. We find here that $SU(5)$ remains unbroken and it is unclear whether hypercharge is broken in the few all-order stringent flat directions calculated. As for the hidden sector, we see from Table 24 that the first and third $SU(2)$'s are broken while the remaining $SU(2)^2 \times SU(4)$ is left unbroken.

	Q_A	ϕ_1	ϕ_2	ϕ_3	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	ψ_6	ψ_7	ψ_8	ψ_9	ψ_{10}	ψ_{11}	ψ_{12}	ψ_{13}	ψ_{14}
$\mathcal{D}_1$	-1	0	0	0	0	0	0	0	0	-3	0	0	0	1	2	2	-2	1
$\mathcal{D}_2$	-1	0	-3	0	0	0	0	0	0	0	0	0	0	-2	2	2	-2	4
$\mathcal{D}_3$	-1	0	0	-3	0	0	0	0	0	0	0	0	0	4	2	2	-2	-2
$\mathcal{D}_4$	0	1	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	2
$\mathcal{D}_5$	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	-1	1
$\mathcal{D}_6$	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	-1
$\mathcal{D}_7$	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	-1	0
$\mathcal{D}_8$	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	1
$\mathcal{D}_9$	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	-1
$\mathcal{D}_{10}$	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	-1
$\mathcal{D}_{11}$	0	0	0	0	0	1	0	0	0	0	0	0	0	-1	1	0	0	0
$\mathcal{D}_{12}$	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1	0	0

Table 9: Basis set of U(1) singlet D-flat directions for the $SO(10)$ model. These directions were sufficient to produce all-order stringent F-flat directions. Column 1 denotes the D-flat direction label. Column 2 is the anomalous charge. The remaining columns specify the norm squared VEVs of the respective non-Abelian singlet fields. All other D-flat tables are structured in a like manner.

FD	Q_A	Φ_1	Φ_2	Φ_3	ϕ_3	ϕ_6	ϕ_8	ϕ_9	ϕ_{10}	ϕ_{11}	ϕ_{12}	ϕ_{13}	ϕ_{14}
1	-49	0	-3	-144	-98	0	0	0	92	98	0	-98	-92
2	-1	0	-1	-2	-2	0	0	0	0	2	0	-2	0
3	-1	0	0	-3	-4	0	0	0	0	2	-2	-2	-2
4	-1	0	-1	0	0	-2	0	-2	0	2	0	-2	0
5	-2	0	-3	-1	0	-2	0	-4	0	4	0	-4	0
6	-1	0	0	1	0	-4	0	-2	0	2	0	-2	0
7	-1	0	0	-1	-2	-2	0	0	0	2	0	-2	0
8	-2	0	-1	-3	-4	-2	0	0	0	4	0	-4	0
9	-1	0	1	0	-2	-4	0	0	0	2	0	-2	0
10	-2	1	0	0	0	-6	0	-4	0	4	0	-4	0
11	-5	1	0	3	0	-18	0	-10	0	10	0	-10	0
12	-2	-1	0	0	-4	-6	0	0	0	4	0	-4	0
13	-5	-1	0	-3	-10	-12	0	0	0	10	0	-10	0
14	-1	0	0	0	0	-3	-2	-3	1	0	-1	-2	0
15	-1	0	0	1	0	-4	-2	-4	0	0	-2	-2	0

Table 10: Example of singlet (i.e. no non-Abelian field VEVs) F-flat directions to all order for the $SO(10)$ model. Column 1 denotes the flat direction number. Column 2 is the anomalous charge. The remaining columns specify the norm squared VEVs of the respective non-Abelian singlet fields. All other F-flat tables are structured in a like manner.

	Q_A	ϕ_1	ψ_1	ϕ_2	ϕ_3	ψ_2	ψ_3	ψ_4	ψ_5	ψ_6	ψ_7	ψ_8	ψ_9	ϕ_4	ϕ_5	ψ_{10}	ψ_{11}	ψ_{12}	ϕ_7	ϕ_6	E_1^c	E_2^c	E_3^c
$\mathcal{D}_1$	0	1	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	2	-2	0	0	0	0	0
$\mathcal{D}_2$	0	0	0	0	0	0	0	0	1	0	0	0	-1	0	0	0	1	0	0	0	0	0	0
$\mathcal{D}_3$	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-1	-1	0	0	0	0	0
$\mathcal{D}_4$	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	-1	-1	0	0	0	0	0
$\mathcal{D}_5$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0
$\mathcal{D}_6$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	2	0	0	0	0	0	0
$\mathcal{D}_7$	0	0	0	0	0	0	0	0	0	0	1	0	-1	0	0	0	1	-1	0	0	0	0	0
$\mathcal{D}_8$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	1	0	0	0
$\mathcal{D}_9$	0	0	0	0	0	1	0	0	0	0	0	0	-1	0	0	0	0	-1	0	0	0	0	0
$\mathcal{D}_{10}$	0	0	0	0	0	0	0	0	0	1	0	0	-1	0	0	0	1	-1	0	0	0	0	0
$\mathcal{D}_{11}$	0	0	0	0	0	0	0	1	0	0	0	0	-1	0	0	0	2	-1	0	0	0	0	0
$\mathcal{D}_{12}$	0	0	0	0	1	0	0	0	0	0	0	0	-2	0	0	0	1	-1	0	0	0	0	0
$\mathcal{D}_{13}$	0	0	0	0	0	0	1	0	0	0	0	0	-1	0	0	0	0	-1	0	0	0	0	0
$\mathcal{D}_{14}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	1	0	0	0	0
$\mathcal{D}_{15}$	0	0	0	1	0	0	0	0	0	0	0	0	-2	0	0	0	0	-2	0	0	0	0	0
$\mathcal{D}_{16}$	0	0	1	0	0	0	0	0	0	0	0	0	-2	0	0	0	1	-2	0	0	0	0	0

Table 11: Basis set of non-anomalous U(1) singlet D-flat directions for flipped $SU(5)$ model #1. Since $Q_A = 0$ for all the D-flat directions, non-Abelian directions needed to be investigated.

	Q_A	H_1	H_2	H_3	F_1	$\bar{F}_1$	F_2	$\bar{F}_2$	F_3	$\bar{F}_3$	H_4	H_{14}	H_{13}	H_{15}	H_7	H_8	H_9
D_1	2	3	-3	-3	0	0	0	0	5	0	0	0	0	0	5	0	0
D_2	2	3	-3	-3	5	0	0	0	0	0	0	0	0	0	0	0	0
D_3	2	3	-3	-3	0	0	5	0	0	0	0	0	0	0	0	5	0
D_4	1	0	0	-2	0	0	0	0	0	0	0	0	0	1	1	1	0
D_5	1	0	-2	0	0	0	0	0	0	0	0	0	1	0	1	0	1
D_6	2	2	-4	-4	0	0	0	0	0	0	1	0	0	0	3	2	2
D_7	4	-9	-1	-1	0	0	0	0	0	5	0	0	0	0	5	0	0
D_8	-6	-9	19	9	0	0	0	5	0	0	0	0	0	0	-10	-5	-10
D_9	-1	0	2	2	0	0	0	0	0	0	0	1	0	0	-2	-1	-1
D_{10}	-6	-9	9	19	0	5	0	0	0	0	0	0	0	0	-10	-10	-5

Table 12: Basis set of non-anomalous U(1) non-Abelian D-flat directions for flipped $SU(5)$ model #1.

	Q_A	ψ_{10}	ψ_{11}	ψ_{12}	E_1^c	E_2^c	E_3^c	H_1	H_2	H_3	H_5	H_6	H_{10}	H_{11}	H_{14}	H_{12}	H_{13}	H_{15}	H_7	H_8	H_9
Δ_1	0	0	0	0	0	0	0	-1	1	1	0	0	0	0	0	1	0	0	-1	0	0
Δ_2	0	0	0	0	0	0	0	-1	1	1	0	0	0	1	0	0	0	0	0	-1	0
Δ_3	0	0	0	0	0	0	0	-1	1	1	0	0	1	0	0	0	0	0	0	0	-1
Δ_4	0	0	0	0	0	0	0	2	0	-2	1	0	0	0	0	0	0	0	0	1	0
Δ_5	0	0	0	0	0	0	0	2	-2	0	0	1	0	0	0	0	0	0	0	0	1
Δ_6	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Δ_7	0	-1	0	1	0	0	0	2	0	-2	0	0	0	0	0	0	0	0	0	0	0
Δ_8	0	0	0	1	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0
Δ_9	1	0	0	0	2	-1	-1	0	0	0	0	0	0	0	0	0	0	3	0	0	0
Δ_{10}	1	0	0	0	-1	-1	2	0	0	0	0	0	0	0	3	0	0	0	0	0	0
Δ_{11}	1	0	0	0	-1	2	-1	0	0	0	0	0	0	0	0	0	3	0	0	0	0

Table 13: Basis set of non-anomalous U(1) mixed singlet/non-Abelian D-flat directions for flipped $SU(5)$ model #1.

FD	Q_A	ψ_9	ψ_{10}	ψ_{11}	$\bar{H}_1$	$\bar{H}_2$	$\bar{H}_3$	H_4	H_{10}	H_{12}	H_7	H_8	H_9
1	-14	-2	6	0	-23	-1	-1	6	1	22	0	16	15
2	-1	-2	6	0	0	-42	-42	15	0	18	31	34	34
3	-14	0	-3	-3	-1	-13	-7	7	0	21	0	14	14
4	-15	0	-3	-3	-1	-15	-9	8	0	23	1	16	16
5	-16	1	-3	0	0	-12	-12	9	0	24	1	16	16
6	-1	0	33	-33	-44	-68	-2	23	0	24	45	46	46
7	-1	0	30	-30	-42	-60	0	21	0	24	39	-42	42
8	-1	0	9	-9	0	-56	-38	19	0	20	37	38	38
9	-14	0	3	-3	-13	-7	-1	7	0	21	0	14	14

Table 14: Example of non-anomalous U(1) F-flat directions for flipped $SU(5)$ model #1. These flat directions required VEVs of non-Abelian fields denoted by H_i and $\bar{H}_i$.

	Q_A	ϕ_1	E_1^c	ψ_1	ψ_2	ψ_3	E_2^c	E_3^c	ψ_4	ψ_5	ψ_6	ψ_7	ψ_8	ψ_9	ψ_{10}	ψ_{11}	ψ_{12}	ψ_{13}	ψ_{14}	ψ_{15}	ψ_{16}	ψ_{17}	ψ_{18}	ψ_{19}	ψ_{20}	ψ_{21}	ψ_{22}	ψ_{23}	ψ_{24}
$\mathcal{D}_1$	-1	0	-5	-3	-3	-2	1	7	-4	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{D}_2$	-1	0	-2	-3	-3	-2	-2	7	-4	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{D}_3$	-1	0	-5	-3	-3	-2	-2	7	-1	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{D}_4$	-1	0	1	-3	-3	1	-2	4	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0
$\mathcal{D}_5$	-1	-3	-2	0	-3	-2	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{D}_6$	1	0	2	0	0	2	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0
$\mathcal{D}_7$	1	0	2	3	3	-4	2	-1	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
$\mathcal{D}_8$	1	0	-1	3	3	-4	2	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0
$\mathcal{D}_9$	1	0	-1	3	3	-4	5	-1	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0
$\mathcal{D}_{10}$	0	0	1	0	0	1	-2	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{D}_{11}$	0	0	-1	-1	0	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
$\mathcal{D}_{12}$	0	0	-1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
$\mathcal{D}_{13}$	0	0	1	0	0	1	0	0	-1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
$\mathcal{D}_{14}$	0	0	1	1	1	1	0	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
$\mathcal{D}_{15}$	0	0	-1	1	0	-1	1	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{D}_{16}$	0	0	0	1	0	1	0	-2	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{D}_{17}$	0	0	-1	1	-1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{D}_{18}$	0	0	-1	-1	-1	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
$\mathcal{D}_{19}$	0	0	-1	0	-2	-1	0	2	-1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{D}_{20}$	0	0	0	-1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
$\mathcal{D}_{21}$	0	0	1	0	0	1	0	-2	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0

Table 15: Basis set of non-anomalous U(1) singlet D-flat directions for flipped $SU(5)$ model #2. This was all that was necessary to find all-order stringent F-flat directions.

FD	Q_A	ϕ_1	E^c_1	ψ_1	ψ_2	ψ_3	E^c_2	E^c_3	ψ_4	ψ_6	ψ_7	ψ_9	ψ_{10}	ψ_{15}	ψ_{19}	ψ_{20}	ψ_{22}
1	-3	0	-4	1	1	4	0	1	0	6	0	2	1	0	0	0	10
2	-3	-6	-4	1	-5	-2	0	1	0	0	0	2	1	0	0	0	4
3	-1	0	-8	1	-3	-2	0	3	0	0	4	4	1	0	2	0	0
4	-2	0	-17	5	-6	-1	0	0	4	0	11	9	1	0	4	0	0
5	-1	0	-8	-7	-3	-2	0	11	0	0	0	4	1	0	2	4	0
6	-2	-3	-10	-3	-6	0	0	8	0	1	0	1	1	4	0	0	0
7	-4	-7	-4	2	-5	-1	0	0	0	1	0	2	2	0	0	0	7
8	-2	-2	-2	1	-1	1	0	0	0	2	0	1	1	0	0	0	5
9	-3	-4	-4	1	3	0	0	1	0	2	0	2	1	0	0	0	6
10	-4	-6	-5	1	-4	0	0	0	1	2	0	3	1	0	0	0	8
11	-3	-1	-4	1	0	3	0	1	0	5	0	2	1	0	0	0	9
12	-4	-2	-5	2	0	4	1	0	0	7	0	2	1	0	0	0	12
13	-4	-2	-5	2	0	4	1	0	0	7	0	2	1	0	0	0	12
14	-4	-7	-11	2	-12	-1	0	0	0	1	7	2	2	0	0	0	0

Table 16: Example of non-anomalous U(1) singlet stringent F-flat directions for flipped $SU(5)$ model #2.

	Q_A	ϕ^V_1	ϕ^V_2	ϕ^V_3	ϕ^V_4	ϕ^V_5	ϕ^V_6	E^c_1	E^c_2	E^c_3	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	ψ_6
$\mathcal{D}_1$	1	0	7	0	-2	-2	1	0	0	0	6	0	0	0	0	6
$\mathcal{D}_2$	1	0	4	-3	-2	1	-2	0	0	0	0	0	6	0	0	6
$\mathcal{D}_3$	1	0	4	-3	1	-2	-2	0	0	0	0	0	0	0	6	6
$\mathcal{D}_4$	0	0	0	0	-1	1	0	0	0	0	0	0	0	2	0	-2
$\mathcal{D}_5$	0	1	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0
$\mathcal{D}_6$	0	0	-1	0	0	0	-1	2	0	-2	0	0	0	0	0	0
$\mathcal{D}_7$	0	0	1	1	-1	0	1	0	0	0	0	2	0	0	0	-2
$\mathcal{D}_8$	0	0	0	0	-1	-1	0	0	2	-2	0	0	0	0	0	0

Table 17: Basis set of non-anomalous U(1) singlet D-flat directions for flipped $SU(5)$ model #3. Since none of the anomalous charges for these directions is negative, non-Abelian directions were investigated.

	Q_a	$\Psi_{1(4)}$	$\Psi_{2(3)}$ $H_{1(1b)}$ H_{28}	$\Psi_{5(8)}$ $H_{2(3)}$ H_{29}	$\Psi_{6(7)}$ $H_{4(5)}$ H_{30}	$\Psi_{9(10)}$ $H_{6(7)}$ H_{31}	$\Psi_{11(12)}$ H_8 H_{32}	Ψ_{13} H_9 H_{33}	Ψ_{14} H_{10} H_{34}	Ψ_{15} H_{11} H_{35}	Ψ_{16} H_{12} H_{36}	Ψ_{17} H_{13} H_{37}	Ψ_{18} H_{14} H_{38}	Ψ_{19} H_{15} H_{39}	Ψ_{20} H_{16} H_{40}	Ψ_{21} H_{17} H_{41}	F_1 H_{18} H_{42}	f_{1b} H_{19} H_{43}	F_2 H_{20} H_{44}	f_{2b} H_{21} H_{45}	F_3 H_{22} H_{46}	f_{3b} H_{23} H_{47}	h_1 H_{24}	h_2 H_{25}	H_{26}	H_{27}
D_1	-1	0	-16	-15	11	-1	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	15		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	12	0	0				
D_2	-1	0	-16	-15	11	14	-7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	15	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	12	0	0				
D_3	1	0	22	21	-11	-2	4	0	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-24	0	0				
D_4	1	0	-2	-15	1	-2	4	0	0	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	24	0	0				
D_5	1	0	-8	-21	1	4	-2	0	0	0	0	0	0	12	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	24	0	0				
D_6	1	0	-4	-11	7	-2	-2	0	0	0	0	0	0	0	0	12	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	24	0	0				
D_7	1	0	46	75	-41	-14	-8	0	0	0	0	0	0	0	0	0	0	0	0	60	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	-36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-36	0	0				
D_8	1	0	10	3	-5	-2	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
D_9	1	0	76	75	-71	-44	22	0	0	0	0	0	0	0	0	0	0	0	0	0	60	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	-72	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-72	0	0				
D_{10}	1	0	-26	-33	19	10	-8	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	24	0	0				
D_{11}	1	0	4	-3	1	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
D_{12}	1	0	4	3	1	4	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	12	0	0	0	0	0	0	0	0				
D_{13}	1	0	-2	-3	1	-2	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0				
D_{14}	1	0	-2	-3	1	-2	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	12	0	0	0	0	0	0	0	0	0	0	0				
D_{15}	1	0	-2	-3	1	-2	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	12	0				

Basis set of non-anomalous $U(1)$ non-Abelian D-flat directions for flipped $SU(5)$ model #3 *continued on next page ...*

. . . continued from previous page

	Q_a	$\Psi_{1(4)}$	$\Psi_{2(3)}$ $H_{1(1b)}$ H_{28}	$\Psi_{5(8)}$ $H_{2(3)}$ H_{29}	$\Psi_{6(7)}$ $H_{4(5)}$ H_{30}	$\Psi_{9(10)}$ $H_{6(7)}$ H_{31}	$\Psi_{11(12)}$ H_8 H_{32}	Ψ_{13} H_9 H_{33}	Ψ_{14} H_{10} H_{34}	Ψ_{15} H_{11} H_{35}	Ψ_{16} H_{12} H_{36}	Ψ_{17} H_{13} H_{37}	Ψ_{18} H_{14} H_{38}	Ψ_{19} H_{15} H_{39}	Ψ_{20} H_{16} H_{40}	Ψ_{21} H_{17} H_{41}	F_1 H_{18} H_{42}	f_{1b} H_{19} H_{43}	F_2 H_{20} H_{44}	f_{2b} H_{21} H_{45}	F_3 H_{22} H_{46}	f_{3b} H_{23} H_{47}	h_1 H_{24}	h_2 H_{25}	H_{26}	H_{27}
D_{16}	1	0	-20	-33	13	4	-2	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	24	0	0				
D_{17}	1	0	-4	-5	-1	6	2	0	0	0	0	0	0	0	0	0	0	0	20	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	0	0				
D_{18}	1	0	4	3	1	4	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
D_{19}	1	0	-4	-5	9	-4	2	0	0	0	0	0	0	0	0	0	0	20	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	0	0				
D_{20}	1	0	-14	-15	9	6	-8	0	0	0	0	0	0	0	0	0	20	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	0	0				
D_{21}	1	0	16	15	-11	4	-2	0	0	0	0	0	12	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-24	0	0				
D_{22}	1	0	4	3	-5	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
D_{23}	1	0	4	3	-5	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
D_{24}	1	0	4	-3	-5	4	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0				
D_{25}	1	0	76	75	-41	-14	22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	60	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	-72	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-72	0	0				
D_{26}	1	0	4	3	1	4	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
D_{27}	1	0	16	15	-5	-2	-2	0	0	0	0	0	0	0	12	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-24	0	0				
D_{28}	1	0	-20	-33	19	10	-2	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	24	0	0				
D_{29}	0	0	-1	-1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0
			0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
D_{30}	0	0	0	1	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	-2	0	0				

Basis set of non-anomalous $U(1)$ non-Abelian D-flat directions for flipped $SU(5)$ model #3 *continued on next page . . .*

. . . continued from previous page

	Q_a	$\Psi_{1(4)}$	$\Psi_{2(3)}$	$\Psi_{5(8)}$	$\Psi_{6(7)}$	$\Psi_{9(10)}$	$\Psi_{11(12)}$	Ψ_{13}	Ψ_{14}	Ψ_{15}	Ψ_{16}	Ψ_{17}	Ψ_{18}	Ψ_{19}	Ψ_{20}	Ψ_{21}	F_1	f_{1b}	F_2	f_{2b}	F_3	f_{3b}	h_1	h_2		
			$H_{1(1b)}$	$H_{2(3)}$	$H_{4(5)}$	$H_{6(7)}$	H_8	H_9	H_{10}	H_{11}	H_{12}	H_{13}	H_{14}	H_{15}	H_{16}	H_{17}	H_{18}	H_{19}	H_{20}	H_{21}	H_{22}	H_{23}	H_{24}	H_{25}	H_{26}	H_{27}
			H_{28}	H_{29}	H_{30}	H_{31}	H_{32}	H_{33}	H_{34}	H_{35}	H_{36}	H_{37}	H_{38}	H_{39}	H_{40}	H_{41}	H_{42}	H_{43}	H_{44}	H_{45}	H_{46}	H_{47}				
D_{31}	0	0	-2	-1	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
D_{32}	0	0	-4	-5	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0
			0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0				
D_{33}	0	0	-3	-3	2	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0				
D_{34}	0	0	-2	-3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0				
D_{35}	0	0	-2	-3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0
			0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0				
D_{36}	0	0	-2	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	2	0	0				
D_{37}	0	0	-3	-4	2	2	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0
			0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0				
D_{38}	0	0	2	2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0
			0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
D_{39}	0	0	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
D_{40}	0	0	3	5	-2	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0
			0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-4	0	0				
D_{41}	0	0	3	4	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0
			0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-4	0	0				
D_{42}	0	0	3	5	-2	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0
			0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-4	0	0				
D_{43}	0	0	1	2	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
			0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0				
D_{44}	0	0	1	1	-1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0
			0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
D_{45}	0	0	-1	-1	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				

Basis set of non-anomalous $U(1)$ non-Abelian D-flat directions for flipped $SU(5)$ model #3 *continued on next page . . .*

. . . continued from previous page

	Q_a	$\Psi_{1(4)}$	$\Psi_{2(3)}$	$\Psi_{5(8)}$	$\Psi_{6(7)}$	$\Psi_{9(10)}$	$\Psi_{11(12)}$	Ψ_{13}	Ψ_{14}	Ψ_{15}	Ψ_{16}	Ψ_{17}	Ψ_{18}	Ψ_{19}	Ψ_{20}	Ψ_{21}	F_1	f_{1b}	F_2	f_{2b}	F_3	f_{3b}	h_1	h_2		
			$H_{1(1b)}$	$H_{2(3)}$	$H_{4(5)}$	$H_{6(7)}$	H_8	H_9	H_{10}	H_{11}	H_{12}	H_{13}	H_{14}	H_{15}	H_{16}	H_{17}	H_{18}	H_{19}	H_{20}	H_{21}	H_{22}	H_{23}	H_{24}	H_{25}	H_{26}	H_{27}
			H_{28}	H_{29}	H_{30}	H_{31}	H_{32}	H_{33}	H_{34}	H_{35}	H_{36}	H_{37}	H_{38}	H_{39}	H_{40}	H_{41}	H_{42}	H_{43}	H_{44}	H_{45}	H_{46}	H_{47}				
D_{46}	0	0	3	4	-2	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
			0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-4	0	0				
D_{47}	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	-2	0	0				
D_{48}	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	-2	0	0				
D_{49}	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0				
D_{50}	0 1	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0			
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
D_{51}	0	0	2	3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-4	0	0				
D_{52}	0	0	-2	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
D_{53}	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
D_{54}	0	0	-1	0	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
D_{55}	0	0	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
			0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
D_{56}	0	0	0	1	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0
			0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
D_{57}	0	0	-1	-1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	2	0	0				
D_{58}	0	0	-1	-2	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	2	0	0				
D_{59}	0	0	-1	-1	2	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
D_{60}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1				

Table 18: Basis set of non-anomalous U(1) non-Abelian D-flat directions for flipped $SU(5)$ model #3. The negative Type I directions listed above are sufficient to produce stringent F-flat directions.

FD	Q_A	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	ψ_6	H_2	H_7	H_9	H_{12}	H_{16}	H_{29}
1	-1	-4	0	2	2	2	-6	0	6	6	0	6	0
2	-1	0	1	3	-2	0	-6	10	6	0	6	0	4
3	-1	-1	0	2	2	-1	-6	6	6	0	0	6	0

Table 19: Example of non-Abelian all-order stringent F-flat directions for flipped $SU(5)$ model #3.

6 Conclusions

In answer to the posits made in [14], we have shown with this sampling of models that reduced Higgs do not correlate to a lack of stringent flat directions. Indeed, it is apparently possible to a find the whole spectrum of possible types of flatness results, some from singlets only, some from non-Abelian only and some from mixed, that also yield a reduced number of Higgs. A summary of each of the models' gauge groups, number of Higgs and number and type of all-order stringent flat directions is presented in Table 20. As can be seen from these results, it is quite possible to find all-order stringent flat directions and still preserve symmetries that can break to the SM gauge group.

Models with flat directions that are all-order stringent flat (or stringent flat beyond 17th order) require an alternative SUSY breaking mechanism. One way to do this is through hidden sector condensates that form at higher energy above the electroweak scale, which requires hidden sector $SU(n)$ or $SO(2n)$ with $n > 3$. While SUSY is broken in the hidden sector at the condensation scale, it can be passed on to MSSM-charged states at the 100 GeV to 1 TeV scale through several alternative processes: gravitational, shadow charges, and gauge kinetic mixing. In each case, weak coupling between the hidden and observable sectors lowers the scale of SUSY breaking among MSSM charged states by (up to) many orders of magnitude.

The next step in the analysis for these models will concern shadow charges. Usually, the observable and hidden sectors are identified by their independent gauge groups, of which their respective matter states are representations. That is, observable fields do not carry hidden sector charge and vice-versa. However, there may be some extra $U(1)$ charges, as seen in these models, resulting from the compactified six dimensions that are common to both sectors. These are known as shadow charges and can be carried by fields from both sectors. There may also be shadow states that carry only shadow charges. Thus, observable and hidden sector states may interact very weakly (at generally very high order) in the superpotential via shadow charge coupling. Hidden sector SUSY breaking from condensates pass to the observable sector through such interactions, but at very suppressed scales. Results concerning hidden sector condensates, their condensation scales and the shadow charge analysis of these and other models is forthcoming.

Observable	Hidden	# Higgs pairs	# flat dir.	Singlet D-flat	NA D–flat
$SO(10)$	$SU(8) \times SU(2)$	2	≥ 15	yes	–
$SU(5) \times U(1)$	$SU(4) \times SU(10)$	0	≥ 9	no	yes
$SU(5) \times U(1)$	$SU(8)$	2+3	≥ 14	yes	–
$SU(5) \times U(1)$	$SU(2)^4 \times SU(4)$	2	≥ 3	no	yes

Table 20: Summary of results showing there is no direct correlation between the number of untwisted reduced Higgs and a lack of all-order stringent flat directions. This table also details whether singlet or non-Abelian D-flat directions were needed.

Acknowledgments

AEF would like to thank the CERN theory division for hospitality. This work was supported by the STFC, by the University of Liverpool and by Baylor University.

A States and Charges of $SO(10)$ and $SU(5)$ Models

State	$SO(10)$	U_A	U_1'	U_2'	U_3'	U_4'	U_5'	$SU(8)$	$SU(2)$
G_1	16	10	0	-2	-4	0	20	1	1
G_2	16	6	2	0	2	0	-30	1	1
G_3	16	12	-2	2	2	0	10	1	1
h_1	10	-12	-2	-2	-2	0	-10	1	1
$\bar{h}_1$	10	12	2	2	2	0	10	1	1
h_2	10	-12	-2	2	-2	0	-10	1	1
$\bar{h}_2$	10	12	2	-2	2	0	10	1	1
Φ_1	1	0	0	0	0	0	0	1	1
Φ_2	1	0	0	0	0	0	0	1	1
Φ_3	1	0	0	0	0	0	0	1	1
$\phi_1\ (\bar{\phi}_1)$	1	0	0	-8	0	0	0	1	1
$\phi_2\ (\bar{\phi}_2)$	1	24	4	4	4	0	20	1	1
$\phi_3\ (\bar{\phi}_3)$	1	24	4	-4	4	0	20	1	1
$\psi_1\ (\bar{\psi}_1)$	1	12	-6	2	2	0	10	1	1
$\psi_2\ (\bar{\psi}_2)$	1	0	2	-2	2	0	-70	1	1
$\psi_3\ (\bar{\psi}_3)$	1	8	-2	2	-10	0	30	1	1
$\psi_4\ (\bar{\psi}_4)$	1	0	0	4	0	0	0	1	1
$\psi_5\ (\bar{\psi}_5)$	1	0	0	4	0	0	0	1	1
$\psi_6\ (\bar{\psi}_6)$	1	24	4	0	4	0	20	1	1
$\psi_7\ (\bar{\psi}_7)$	1	12	-6	-2	2	0	10	1	1
$\psi_8\ (\bar{\psi}_8)$	1	0	2	2	2	0	-70	1	1
$\psi_9\ (\bar{\psi}_9)$	1	8	-2	-2	-10	0	30	1	1
$\psi_{10}\ (\bar{\psi}_{10})$	1	-20	2	-2	2	16	-54	1	1
$\psi_{11}\ (\bar{\psi}_{11})$	1	-20	0	0	0	16	16	1	1
$\psi_{12}\ (\bar{\psi}_{12})$	1	12	0	0	8	-16	24	1	1
$\psi_{13}\ (\bar{\psi}_{13})$	1	-8	-4	0	4	16	-44	1	1
$\psi_{14}\ (\bar{\psi}_{14})$	1	-20	2	2	2	16	-54	1	1
$H_1\ (\bar{H}_1)$	1	24	0	0	0	8	48	1	2
$H_2\ (\bar{H}_2)$	1	0	-4	4	-4	8	28	1	2
$H_3\ (\bar{H}_3)$	1	0	-4	-4	-4	8	28	1	2
$H_4\ (\bar{H}_4)$	1	-12	2	2	-6	8	18	1	2
$H_5\ (\bar{H}_5)$	1	0	-2	2	-2	8	-42	1	2
$H_6\ (\bar{H}_6)$	1	-8	-2	2	6	8	-2	1	2
$H_7\ (\bar{H}_7)$	1	0	-4	0	-4	8	28	1	2

$SO(10)$ model continued on next page ...

... $SO(10)$ *model continued from previous page*									
State	$SO(10)$	U_A	U_1'	U_2'	U_3'	U_4'	U_5'	$SU(8)$	$SU(2)$
H_8 $(\bar{H}_8)$	1	-12	2	-2	-6	8	18	1	2
H_9 $(\bar{H}_9)$	1	0	-2	-2	-2	8	-42	1	2
H_{10} $(\bar{H}_{10})$	1	-8	-2	-2	6	8	-2	1	2
H_{11} $(\bar{H}_{11})$	1	0	0	-2	-4	-8	-28	8	1
H_{12} $(\bar{H}_{12})$	1	2	-2	2	2	-8	-38	8	1
H_{13} $(\bar{H}_{13})$	1	10	0	0	0	8	48	8	1

Table 21: States of the $SO(10)$ model and their gauge charges. The names of the states appear in the first column, with the states' various charges appearing in the other columns. All $U(1)$ charges are multiplied by a factor of 4 (and similarly for all other models).

State	$SU(5)$	U_A	U_1'	U_2'	U_3'	U_4'	U_5'	U_6'	$SU(4)$	$SO(10)$
F_1	10	6	-2	-2	0	2	-6	2	1	1
F_2	10	6	2	0	-4	-2	0	2	1	1
F_3	10	6	0	2	2	4	6	2	1	1
$\bar{f}_1$	$\bar{5}$	2	-2	2	0	-6	-2	-6	1	1
$\bar{f}_2$	$\bar{5}$	2	2	0	4	-2	-8	-6	1	1
$\bar{f}_3$	$\bar{5}$	2	0	-2	2	-4	10	-6	1	1
E_1^c	1	2	-2	2	0	-6	-2	10	1	1
E_2^c	1	2	2	0	4	-2	-8	10	1	1
E_3^c	1	2	0	-2	2	-4	10	10	1	1
Φ_1	1	0	0	0	0	0	0	0	1	1
Φ_2	1	0	0	0	0	0	0	0	1	1
Φ_3	1	0	0	0	0	0	0	0	1	1
$\phi_1(\bar{\phi}_1)$	1	-8	0	0	0	-16	8	0	1	1
$\phi_2(\bar{\phi}_2)$	1	-8	0	-4	8	-8	-4	0	1	1
$\phi_3(\bar{\phi}_3)$	1	-8	0	4	8	-8	-4	0	1	1
$\phi_4(\bar{\phi}_4)$	1	0	0	-8	0	0	0	0	1	1
$\phi_5(\bar{\phi}_5)$	1	0	0	4	-8	-8	12	0	1	1
ϕ_6	1	0	0	-4	-8	-8	12	0	1	1
ϕ_7	1	0	0	-4	8	8	-12	0	1	1
$\psi_1(\bar{\psi}_1)$	1	-8	0	-2	4	-12	2	0	1	1
$\psi_2(\bar{\psi}_2)$	1	-4	0	-2	4	-4	-2	0	1	1
$\psi_3(\bar{\psi}_3)$	1	-4	0	-2	4	-4	-2	0	1	1
										$SU(5)$ *model #1 continued on next page ...*

... $SU(5)$ *model #1 continued from previous page*

State	$SU(5)$	U_A	U_1'	U_2'	U_3'	U_4'	U_5'	U_6'	$SU(4)$	$SO(10)$
$\psi_4(\bar{\psi}_4)$	1	-4	0	2	-4	-12	10	0	1	1
$\psi_5(\bar{\psi}_5)$	1	-4	0	6	4	-4	-2	0	1	1
$\psi_6(\bar{\psi}_6)$	1	-4	0	0	0	-8	4	0	1	1
$\psi_7(\bar{\psi}_7)$	1	-4	0	0	0	8	4	0	1	1
$\psi_8(\bar{\psi}_8)$	1	-4	0	-4	8	0	-8	0	1	1
$\psi_9(\bar{\psi}_9)$	1	-4	0	4	8	0	-8	0	1	1
$\psi_{10}(\bar{\psi}_{10})$	1	0	0	-2	4	4	-6	0	1	1
$\psi_{11}(\bar{\psi}_{11})$	1	0	0	-2	4	4	-6	0	1	1
$\psi_{12}(\bar{\psi}_{12})$	1	0	0	-6	-4	-4	6	0	1	1
$H_1(\bar{H}_1)$	1	-8	0	-2	-1	-2	2	-5	$\bar{4}$	1
$H_2(\bar{H}_2)$	1	-4	0	0	-5	2	4	-5	$\bar{4}$	1
$H_3(\bar{H}_3)$	1	-4	0	-2	-1	6	-2	-5	$\bar{4}$	1
H_4	1	2	0	-2	-3	6	10	5	$\bar{4}$	1
H_5	1	2	2	0	-1	8	-8	5	$\bar{4}$	1
H_6	1	2	-2	2	-5	4	-2	5	$\bar{4}$	1
H_7	1	6	0	-2	-5	2	-6	-5	4	1
H_8	1	6	-2	0	1	8	0	-5	4	1
H_9	1	6	2	2	-3	4	6	-5	4	1
H_{10}	1	6	2	2	2	-6	6	0	6	1
H_{11}	1	6	-2	0	6	-2	0	0	6	1
H_{12}	1	6	0	-2	0	-8	-6	0	6	1
H_{13}	1	10	-2	0	-2	-2	8	0	1	10
H_{14}	1	10	0	2	0	0	-10	0	1	10
H_{15}	1	10	2	-2	2	2	2	0	1	10

Table 22: States of flipped $SU(5)$ model #1 and their charges.

State	$SU(5)$	U_A	U_1'	U_2'	U_3'	U_4'	U_5'	U_6'	U_7'	$SU(8)$
F_1	10	20	2	-2	2	-2	-2	-28	0	1
F_2	10	24	0	0	0	2	16	-104	0	1
F_3	10	20	2	2	-2	-2	-2	-28	0	1
$\bar{f}_1$	$\bar{5}$	-20	-6	-2	2	-2	-34	-148	0	1
$\bar{f}_2$	$\bar{5}$	-24	-8	0	0	-2	-16	104	0	1
$\bar{f}_3$	$\bar{5}$	-24	-6	2	2	2	-34	16	0	1
E_1^c	1	60	10	-2	2	-2	30	92	0	1
$\bar{E}_1^c$	1	-60	-10	2	-2	2	-30	-92	0	1

$SU(5)$ *model #2 continued on next page ...*

... $SU(5)$ model #2 continued from previous page										
State	$SU(5)$	U_A	U_1'	U_2'	U_3'	U_4'	U_5'	U_6'	U_7'	$SU(8)$
E_2^c	1	56	8	0	0	-2	48	344	0	1
E_3^c	1	56	10	2	2	2	30	256	0	1
$h_1(\bar{h}_1)$	5	-4	-4	4	0	0	-36	-12	0	1
$h_2(\bar{h}_2)$	5	12	0	-2	-2	-2	8	-52	-8	1
h_3	5	20	6	2	-2	2	34	148	0	1
h_4	5	8	0	2	-2	2	8	112	8	1
h_5	5	8	0	-2	2	2	8	112	-8	1
h_6	$\bar{5}$	-24	-6	-2	-2	2	-34	16	0	1
h_7	$\bar{5}$	8	4	0	0	2	36	-152	0	1
h_8	$\bar{5}$	32	4	0	0	-2	-4	272	0	1
Φ_1	1	0	0	0	0	0	0	0	0	1
Φ_2	1	0	0	0	0	0	0	0	0	1
Φ_3	1	0	0	0	0	0	0	0	0	1
$\phi_1(\bar{\phi}_1)$	1	36	-4	-4	0	0	-4	108	0	1
$\psi_1(\bar{\psi}_1)$	1	44	4	2	2	2	4	220	-8	1
$\psi_2(\bar{\psi}_2)$	1	4	0	4	0	2	0	-164	0	1
$\psi_3(\bar{\psi}_3)$	1	-4	0	4	0	-2	0	164	0	1
ψ_4	1	56	10	-2	-2	2	30	256	0	1
ψ_5	1	12	8	2	2	-2	8	-52	8	1
ψ_6	1	12	8	-2	-2	-2	8	-52	-8	1
ψ_7	1	16	6	0	4	2	26	-128	8	1
ψ_8	1	20	6	0	0	-2	26	-292	8	1
ψ_9	1	12	6	0	0	-6	26	36	-8	1
ψ_{10}	1	8	6	0	-4	-2	26	200	-8	1
ψ_{11}	1	8	0	4	-4	-2	0	-328	0	1
ψ_{12}	1	0	0	4	4	6	0	0	0	1
ψ_{13}	1	0	-4	0	0	-2	36	176	0	1
ψ_{14}	1	0	0	-4	-4	6	0	0	0	1
ψ_{15}	1	8	0	-4	4	-2	0	-328	0	1
ψ_{16}	1	40	-4	0	0	2	-4	-56	0	1
ψ_{17}	1	-12	-4	2	2	-2	-44	-124	-8	1
ψ_{18}	1	-48	-4	2	-2	2	-4	-56	-8	1
ψ_{19}	1	-12	-4	-2	-2	-2	-44	-124	8	1
ψ_{20}	1	-8	-6	0	4	2	-26	-200	-8	1
ψ_{21}	1	-4	-6	0	0	-2	-26	-364	-8	1
ψ_{22}	1	-48	-4	-2	2	2	-4	-56	8	1
ψ_{23}	1	-12	-6	0	0	-6	-26	-36	8	1
$SU(5)$ model #2 continued on next page ...										

... $SU(5)$ model #2 continued from previous page

State	$SU(5)$	U_A	U_1'	U_2'	U_3'	U_4'	U_5'	U_6'	U_7'	$SU(8)$
ψ_{24}	1	-16	-6	0	-4	-2	-26	128	8	1
H_1	1	32	4	2	2	-2	24	8	-4	8
H_2	1	32	6	0	0	2	6	-80	-4	8
H_3	1	16	-2	-2	2	2	-2	136	4	8
H_4	1	20	0	0	0	2	-20	-116	4	8
H_5	1	16	-2	2	-2	2	-2	136	4	8
H_6	1	20	-2	2	2	-2	-2	-28	4	8
H_7	1	-28	-4	2	-2	-2	-24	-172	-4	8

Table 23: States of flipped $SU(5)$ model #2 and their charges.

State	$SU(5)$	U_A	U_1'	U_2'	U_3'	U_4'	U_5'	U_6'	U_7'	$SU(2)^4 \times SU(4)$
F_1	10	8	0	2	-2	0	-2	-4	-2	(1, 1, 1, 1, 1)
F_2	10	6	0	2	2	4	0	2	-6	(1, 1, 1, 1, 1)
F_3	10	6	0	2	0	-2	4	2	12	(1, 1, 1, 1, 1)
$\bar{f}_1$	$\bar{5}$	8	0	-6	-2	0	-2	-4	-2	(1, 1, 1, 1, 1)
$\bar{f}_2$	$\bar{5}$	2	0	-6	2	-4	0	-2	-14	(1, 1, 1, 1, 1)
$\bar{f}_3$	$\bar{5}$	2	0	-6	0	-2	-4	6	4	(1, 1, 1, 1, 1)
E_1^c	1	8	0	10	-2	0	-2	-4	-2	(1, 1, 1, 1, 1)
E_2^c	1	2	0	10	2	-4	0	- 2	-14	(1, 1, 1, 1, 1)
E_3^c	1	2	0	10	0	-2	-4	6	4	(1, 1, 1, 1, 1)
$h_1(\bar{h}_1)$	$\bar{5}$	-8	0	4	-4	0	0	0	20	(1, 1, 1, 1, 1)
$h_2(\bar{h}_2)$	$\bar{5}$	-8	0	4	4	0	0	0	20	(1, 1, 1, 1, 1)
Φ_1	1	0	0	0	0	0	0	0	0	(1, 1, 1, 1,1)
Φ_2	1	0	0	0	0	0	0	0	0	(1, 1, 1, 1, 1)
Φ_4	1	0	0	0	0	0	0	0	0	(1, 1, 1, 1, 1)
$\phi_1(\bar{\phi}_1)$	1	4	0	0	0	0	-12	-4	8	(1, 1, 1, 1, 1)
$\phi_2(\bar{\phi}_2)$	1	12	0	0	0	0	4	-12	24	(1, 1, 1, 1, 1)
$\phi_3(\bar{\phi}_3)$	1	-8	0	0	0	-8	-8	0	-16	(1, 1, 1, 1, 1)
$\phi_4(\bar{\phi}_4)$	1	0	0	0	0	-8	8	-8	0	(1, 1, 1, 1, 1)
$\phi_5(\bar{\phi}_5)$	1	0	0	0	4	4	0	-8	-36	(1, 1, 1, 1, 1)
$\phi_6(\bar{\phi}_6)$	1	0	0	0	-4	4	0	-8	-36	(1, 1, 1, 1, 1)
ψ_1	1	4	8	0	2	-2	2	8	-10	(1, 1, 1, 1, 1)
ψ_2	1	4	-8	0	2	-2	2	8	-10	(1, 1, 1, 1, 1)
ψ_3	1	6	8	0	-2	-6	0	2	-6	(1, 1, 1, 1, 1)

$SU(5)$ model #3 continued on next page ...

... $SU(5)$ *model #3 continued from previous page*

State	$SU(5)$	U_A	U_1'	U_2'	U_3'	U_4'	U_5'	U_6'	U_7'	$SU(2)^4 \times SU(4)$
ψ_4	1	6	-8	0	-2	-6	0	2	-6	(1, 1, 1, 1, 1)
ψ_5	1	6	8	0	0	0	-4	2	-24	(1, 1, 1, 1, 1)
ψ_6	1	6	-8	0	0	0	-4	2	-24	(1, 1, 1, 1, 1)
$H_1(\bar{H}_1)$	1	0	8	0	0	0	0	0	0	(1, 2, 2, 1, 1)
H_2	1	-2	4	5	-2	5	-4	2	-4	(1, 1, 2, 1, 1)
H_3	1	2	-4	-5	2	-5	4	-2	4	(1, 1, 1, 2, 1)
H_4	1	4	0	0	2	-2	2	8	-10	(1, 1, 1, 1, 6)
H_5	1	-4	0	0	-2	2	-2	-8	10	(1, 2, 1, 2, 1)
H_6	1	10	0	0	-2	2	0	6	2	(1, 1, 1, 1, 6)
H_7	1	-10	0	0	2	-2	0	-6	-2	(2, 1, 2, 1, 1)
H_8	1	-4	4	5	0	3	2	8	10	(1, 1, 2, 1, 1)
H_9	1	4	4	5	0	3	-2	0	26	(1, 1, 2, 1, 1)
H_{10}	1	2	4	5	-2	5	4	-2	4	(1, 1, 2, 1, 1)
H_{11}	1	2	-4	5	-2	5	4	-2	4	(2, 1, 1, 1, 1)
H_{12}	1	-4	-4	5	2	1	6	0	-8	(2, 1, 1, 1, 1)
H_{13}	1	4	4	5	2	1	2	-8	8	(1, 1, 1, 2, 1)
H_{14}	1	-2	4	5	-2	-3	4	-6	-4	(1, 1, 1, 2, 1)
H_{15}	1	-2	4	5	4	-1	0	2	14	(1, 1, 1, 2, 1)
H_{16}	1	-2	-4	5	-4	-1	0	2	14	(2, 1, 1, 1, 1)
H_{17}	1	-4	-4	5	2	1	6	0	-8	(1, 2, 1, 1, 1)
H_{18}	1	0	-4	5	2	1	-6	-4	0	(1, 2, 1, 1, 1)
H_{19}	1	0	-4	-5	0	5	2	4	-18	(1, 1, 2, 1, 1)
H_{20}	1	8	-4	-5	0	5	-2	-4	-2	(1, 1, 2, 1, 1)
H_{21}	1	6	4	-5	2	3	4	2	12	(1, 2, 1, 1, 1)
H_{22}	1	2	4	-5	2	3	-4	6	4	(1, 2, 1, 1, 1)
H_{23}	1	6	4	-5	2	3	4	2	12	(2, 1, 1, 1, 1)
H_{24}	1	8	4	-5	-2	-1	2	-4	16	(2, 1, 1, 1, 1)
H_{25}	1	0	-4	-5	-2	-1	6	4	0	(1, 1, 1, 2, 1)
H_{26}	1	2	-4	-5	4	1	0	-2	-14	(1, 1, 1, 2, 1)
H_{27}	1	2	4	-5	-4	1	0	-2	-14	(2, 1, 1, 1, 1)
H_{28}	1	0	-4	-5	-2	-1	6	4	0	(1, 1, 2, 1, 1)
H_{29}	1	4	-4	-5	-2	-1	-6	0	8	(1, 1, 2, 1, 1)
H_{30}	1	10	0	0	0	0	4	-2	-16	(1, 2, 2, 1, 1)
H_{31}	1	6	0	0	0	0	-4	2	-24	(2, 1, 1, 2, 1)
H_{32}	1	10	0	0	0	0	4	-2	-16	(1, 1, 1, 1, 6)
H_{33}	1	10	0	0	-2	2	0	6	2	(2, 1, 1, 2, 1)
H_{34}	1	6	0	0	-2	-6	0	2	-6	(1, 2, 2, 1, 1)

$SU(5)$ *model #3 continued on next page ...*

. . . SU(5) model #3 continued from previous page										
State	$SU(5)$	U_A	U_1'	U_2'	U_3'	U_4'	U_5'	U_6'	U_7'	$SU(2)^4 \times SU(4)$
H_{35}	1	12	0	0	2	-2	-2	0	6	(2, 1, 1, 2, 1)
H_{36}	1	12	0	0	2	-2	-2	0	6	(1, 2, 2, 1, 1)
H_{37}	1	2	-4	0	2	2	0	6	22	(1, 1, 1, 1, 4)
H_{38}	1	-2	-4	0	-2	-2	0	-6	-22	(1, 1, 1, 1, 4)
H_{39}	1	10	0	0	-2	2	0	6	2	(1, 2, 1, 2, 1)
H_{40}	1	4	4	0	-2	-2	-2	0	26	(1, 1, 1, 1, $\bar{4}$)
H_{41}	1	4	4	0	2	2	-2	-8	-10	(1, 1, 1, 1, $\bar{4}$)
H_{42}	1	8	-4	0	0	4	2	-4	16	(1, 1, 1, 1, 4)
H_{43}	1	4	4	0	0	4	-6	0	8	(1, 1, 1, 1, $\bar{4}$)
H_{44}	1	4	-4	0	0	-4	2	-8	8	(1, 1, 1, 1, 4)
H_{45}	1	0	4	0	0	-4	-6	-4	0	(1, 1, 1, 1, $\bar{4}$)
H_{46}	1	12	0	0	2	-2	-2	0	6	(2, 1, 2, 1, 1)
H_{47}	1	0	0	0	0	0	0	0	0	(2, 1, 1, 2, 6)

Table 24: States of flipped $SU(5)$ model #3 and their charges.

References

[1] P. Candelas, G. Horowitz, A. Strominger and E. Witten, Nucl. Phys. B **258** (1985) 46.

[2] I. Antoniadis, J. Ellis, J. Hagelin and D. Nanopoulos Phys. Lett. B **231** (1989) 65;
J. Lopez, D. Nanopoulos and K. Yuan, Nucl. Phys. B **399** (1993) 3.

[3] A. Faraggi, D. Nanopoulos and K. Yuan, Nucl. Phys. B **335** (1990) 347;
A. Faraggi, Phys. Rev. D **46** (1992) 3204.

[4] I. Antoniadis, G. Leontaris and J. Rizos, Phys. Lett. B **245** (1990) 161;
G. Leontaris and J. Rizos, Nucl. Phys. B **554** (1999) 3.

[5] A. Faraggi, Phys. Lett. B **278** (1992) 131; Nucl. Phys. B **403** (1993) 101; Nucl. Phys. B **407** (1993) 57;

[6] A. Faraggi, Phys. Lett. B **274** (1992) 47; Phys. Rev. D **47** (1993) 5021; Phys. Lett. B **377** (1996) 43; Nucl. Phys. B **487** (1997) 55.

[7] G. Cleaver, A. Faraggi and D. Nanopoulos, Phys. Lett. B **455** (1999) 135; *Int. J. Mod. Phys.* **A16** (2001) 425;
G. Cleaver, A. Faraggi, D. Nanopoulos and J. Walker, Nucl. Phys. B **593** (2001) 471.

[8] G. Cleaver, A. Faraggi and C. Savage, Phys. Rev. D **63** (2001) 066001;
G. Cleaver, D. Clements and A. Faraggi, Phys. Rev. D **65** (2002) 106003.

[9] B. Assel *et al*, Phys. Lett. B **683** (2010) 306; Nucl. Phys. B **844** (2011) 365.

[10] A. Faraggi, Phys. Lett. B **326** (1994) 62; hep-th/9511093;
J. Ellis, A. Faraggi and D. Nanopoulos, Phys. Lett. B **419** (1998) 123;
P. Berglund, J. Ellis, A. Faraggi, D. Nanopoulos and Z. Qiu, Phys. Lett. B **433** (1998) 269; *Int. J. Mod. Phys.* **A15** (2000) 1345;
A. Faraggi, Phys. Lett. B **544** (2002) 207; hep-th/0411118;
A. Faraggi and R. Donagi, Nucl. Phys. B **694** (2004) 187;
A. Faraggi, S. Förste, M. Timirgaziu, JHEP **0**608, (2006) 057.

[11] G. Cleaver, A. Faraggi, D. Nanopoulos, and J. Walker, *Mod. Phys. Lett.* **A15** (2000) 1191; G. Cleaver, *Int. J. Mod. Phys.* **A16S1C** (2001) 949.

[12] A. Faraggi, Nucl. Phys. B **428** (1994) 111; Phys. Lett. B **520** (2001) 337.

[13] A. Faraggi, E. Manno and C. Timirgaziu, Eur. Phys. Jour. C **50** (2007) 701.

[14] G. Cleaver, A. Faraggi, E. Manno and C. Timirgaziu, Phys. Rev. D **78** (2008) 046009.

[15] A. Faraggi and D. Nanopoulos, Phys. Rev. D **48** (1993) 3288;
A. Faraggi, Nucl. Phys. B **387** (1992) 239; *Int. J. Mod. Phys.* **A14** (1999) 1663.

[16] I. Antoniadis, C. Bachas, and C. Kounnas, Nucl. Phys. B **289** (1987) 87;
H. Kawai, D. Lewellen, and S. Tye, Nucl. Phys. B **288** (1987) 1.

[17] S. Kalara, J. Lopez and D. Nanopoulos, Nucl. Phys. B **353** (1991) 650.

[18] For review and references, see *e.g.*: A. Faraggi, *Int. J. Mod. Phys.* **A19** (2004) 5523; hep-ph/9707311; arXiv:1002.0052.

[19] G. Cleaver and A. Faraggi, *Int. J. Mod. Phys.* **A14** (1999) 2335

[20] M. Green and J. Schwarz Phys. Lett. B **149** (1984) 117;
M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B **289** (1987) 585;
J. Atick, L. Dixon and A. Sen, Nucl. Phys. B **292** (87) 109.

[21] A. Font, L. Ibanez, H. Nilles and F. Quevedo, Nucl. Phys. B **307** (1988) 109;
J. Casas, E. Katehou and C. Munoz, Nucl. Phys. B **317** (1989) 171;
T. Kobayashi and H. Nakano, Nucl. Phys. B **496** (1997) 103;
M. Gaillard and J. Giedt, Phys. Lett. B **479** (2000) 308.

[22] F. Buccella, J. Derendinger, S. Ferrara and C. Savoy, Phys. Lett. B **115** (1982) 375.

[23] G. Cleaver, M. Cvetic, J. Espinosa, L. Everett, and P. Langacker, Nucl. Phys. B **525** (1998) 3; Nucl. Phys. B **545** (1999) 47.

[24] G. Cleaver, *Mass Hierarchy and flat directions in string models*, In Miami Beach 1997, Physics of Mass, 1997, 101.

[25] G. Cleaver, A. Faraggi, D. Nanopoulos and J. Walker, Nucl. Phys. B **620** (2002) 259.

[26] E. Witten, Nucl. Phys. B **202** (1982) 253.

[27] See *e.g.*: K. Intriligator and N. Seiberg, hep-ph/0702069.

[28] G. Cleaver, arXiv:hep-ph/0703027.

[29] A. Faraggi and E. Halyo, *Int. J. Mod. Phys.* **A11** (1996) 2357.

[30] A. Faraggi, Nucl. Phys. B **728** (2005) 83.

[31] W. Buchmuller, K. Hamaguchi, O. Lebedev and M. Ratz, arXiv:hep-th/0606187.

[32] J. Wess and J. Bagger, *Supersymmetry and supergravity*, http://www.slac.stanford.edu/spires/find/hep/www?irn=5426545 (SPIRES entry).

[33] Press, et al., *Numerical Recipes*, (Cambridge, England: Cambridge University Press, 1989).

[34] G. Cleaver, J. Ellis and D. Nanopoulos, Nucl. Phys. B **600** (2001) 315.

[35] G. Cleaver, A. Faraggi and S. Nooij, Nucl. Phys. B **672** (2003) 64.

[36] J. Perkins *et al.*, Phys. Rev. D **75** (2007) 026007.

[37] N. Krasnikov, Phys. Lett. B **193** (1987) 37;
L. Dixon, SLAC–PUB–5229, invited talk given at 15th APS Div. of Particles and Fields General Mtg., Houston,TX, Jan 3-6, 1990. Published in DPF Conf.1990:811-822;
J.A. Casas, Z. Lalak, C. Munoz and G. Ross, Nucl. Phys. B **347** (1990) 243.